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A MATHEMATICAL MODEL OF DIFFERENTIAL TRACHEAL TUBE CUFF PRESSURE: EFFECTS OF DIFFUSION AND TEMPERATURE

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ABSTRACT. The tracheal tube cuff performs an important function during anesthesia and critical care situations by allowing positive pressure ventilation and isolating the lungs from aspiration. Other maneuvers, such as pressure support ventilation and positive end-expiratory pressure, are also cuff-dependent. However, excessive cuff pressure, as well as long-term intubation without excessive cuff pressure, have been associated with significant morbidity and mortality. A straightforward mathematical model of differential tracheal tube cuff pressure has been developed. This model incorporates compliance, temperature variation, and net molar diffusion in determining differential tracheal tube cuff pressure. In addition, temperature and diffusion are modeled as separate processes which effect differential cuff pressure independently. Support for the validity of this model is based upon an analysis of existing data from prior studies.

KEY WORDS. tracheal tube, nitrous oxide, diffusion, temperature, anesthesia, critical care, trachea.

INTRODUCTION

Excessive tracheal tube cuff pressure has been reported as a causative factor in: tracheal-innominate artery fistula, tracheal-esophageal fistula, tracheal stricture and vocal cord damage. Long-term intubation, without excessive cuff pressure, has also been associated with these, and other, pathologies [1–12].

Studies have examined the large increase in tracheal tube cuff pressure that occurs primarily from the diffusion of nitrous oxide into the cuff [13, 14]. Extreme changes in body temperature, occurring with anatomic changes during cardiopulmonary bypass, have also been associated with changes in cuff pressure [15–17].

Increases in temperature, of the gases within the tracheal tube cuff, appear to have a small but consistent effect of increasing cuff pressure. This temperature-related increase in cuff pressure can be modeled independently of the diffusion process and occurs as cuff gases warm from room to body temperature.

In addition, the compliance of the cuff varies considerably between manufacturers and materials [14]. Depending on the gases within the cuff, as well as the inhaled anesthetic and ventilation gases, nitrous oxide may be diffusing into the cuff at a rate faster than air diffusing out [18–21].

Therefore, a mathematical model of tracheal tube cuff pressure would provide useful insight into the design and development of cuffs for tracheal tubes as well as for tracheostomy devices. This model could also be applied to specialized tracheal tube cuffs. Examples of these would

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Fig. 1. Various tracheal tubes, with their cuffs inflated, are illustrated. From top to bottom: plain, oral reverse angle endotracheal (rae), double cuff tracheal tube for endobronchial laser surgery, and a double lumen tracheal tube for single-lung ventilation.

include tracheal tube cuffs for single-lung ventilation as well as those for laser surgery within the trachea and bronchi. In addition, reverse angle endotracheal (rae) tubes, which are used for oral, facial, and nasal surgery, have cuffs with a noticeably different shape and compliance.

Various tracheal tubes, with different cuffs, are shown in Figure 1.

Volume, pressure, and compliance

The typical tracheal tube cuff is manufactured from a highly compliant polyvinylchloride plastic [14]. Recalling that compliance is defined as dV/dP or a change in volume with respect to a change in pressure:

$$C = \frac{dV}{dP}. \tag{1}$$

Therefore, volume, as a function of pressure and compliance, can be expressed as:

$$V = \int dV = C \int dP = CP + A. \tag{2}$$

where A represents the *constant of integration*.

It should be noted that the linear equation for volume: $V = CP + A$ is a reasonable approximation when the tracheal tube cuff is inflated under conditions of normal, and near normal, pressure.

The constant of integration, A, can be readily determined from initial conditions:

$$A = V_i - CP_i. \tag{3}$$

where V_i and P_i refer to the initial cuff volume and pressure respectively.

Clinically, room air, *at room temperature*, is injected into the tracheal tube cuff following laryngoscopy and intubation. When anesthetic gases are used to inflate the cuff, these are also at room temperature.

A seal is then formed, between the cuff and the trachea, allowing positive pressure ventilation and preventing the leakage of ventilation gases and inhalational anesthetic agents. This seal also helps to prevent aspiration. Thus, substantial amounts, of regurgitated gastric contents and oral secretions, are usually isolated from the lungs and trachea.

However, even on a short-term basis, cuff pressures exceeding a value of approximately 20 mmHg have been associated with significant acute and chronic pathological changes in the tracheal mucosa [4, 11, 18, 22].

It should be noted that a more compliant cuff will have a constant of integration, A, with a *greater negative value* than a cuff which is less compliant. This is illustrated in Figure 2. Furthermore, this concept is verified from the analysis of the clinical data in Table 1 and is shown graphically in Figure 3.

In the absence of overinflation, cuff compliance varies widely from approximately 0.12 to 0.28 ml/mmHg. While the associated constants of integration range from −87.8 to −212.3 ml respectively. These data are shown in Table 1.

In addition, cuff compliance has been noted to vary between *in-vivo* and *in-vitro* examinations [14, 21, 23].

The ideal gas equation

The ideal gas equation expresses the relationship of pressure, volume, and temperature for a molar quantity of gas in a *fixed* volume:

$$PV = nRT. \tag{4}$$

Digressing from clinical pressure, relative pressure, P_{rel}, must be converted to absolute atmospheric pressure in mmHg. The conversion is simply: $P = P_{\mathrm{rel}} + 760\,\mathrm{mmHg}$. Temperature is also converted to absolute units and expressed in degrees Kelvin: $T = T\,^{\circ}\mathrm{C} + 273.15$ where $^{\circ}\mathrm{C}$ is temperature in degrees Celsius.

R, the gas constant, is based on units of absolute pressure in mmHg, absolute temperature in degrees Kelvin, and volume in milliliters: $R = 6.236 \times 10^4\ \mathrm{ml\,mmHg\,mol^{-1}\,K^{-1}}$.

Table 1. Changes in tracheal tube cuff pressure can be readily estimated utilizing compliance, the change in the number of moles within the cuff, and assuming ΔT is to be body minus room temperature ($37 - 20 = 17\,^{\circ}C$). ΔP_{moles} is the contribution to cuff pressure change from diffusion, whereas ΔP_{Temp} is the contribution to cuff pressure change from temperature

	Profile Soft-Seal	Hi-Contour	Reinforced	Sheridan	Trachelon	NCC Hi-Lo	NCC rae	NCC plain	Portex blue line	Equation
Cuff Pressure (mmHg)										
Initial	10.8 (2.4)	9.7 (2.4)	9.3 (1.5)	10.4 (1.2)	9.5 (1.1)	10 (3)	11.7	12.5	12.3	
Final	12.7 (5.3)	11.0 (3.6)	10.9 (2.9)	12.7 (3.8)	13.3 (4.2)	52 (13)	23.4	26.5	23.2	
Measured ΔP (mmHg)	**1.9 (5.3)**	**1.3 (4.4)**	**1.6 (2.8)**	**2.3 (3.8)**	**3.7 (4.3)**	**42 (11)**	**11.7**	**14**	**10.9**	
Cuff Volume (ml)										
Initial	4.5 (1.9)	6.0 (2.5)	4.8 (1.7)	6.6 (2.1)	5.1 (0.7)	6 (2)	3.8	4.9	5.7	
Final	4.9 (1.5)	6.3 (2.1)	5.1 (2.2)	7.2 (2.5)	5.7 (1.0)	14 (4)	7.1	6.6	7.1	
Compliance (ml/mmHg)	0.21	0.23	0.19	0.26	0.16	0.19	0.28	0.12	0.13	Equation (1)
Constant of integration; A (ml)	−158.1	−171.8	−141.4	−194.5	−118	−140.3	−212.3	−87.8	−94.7	Equation (3)
dP/dn	1.094×10^5	9.946×10^4	1.211×10^5	8.803×10^4	1.426×10^5	1.2×10^5	8.314×10^4	1.873×10^5	1.723×10^5	Equation (16)
dP/dT	0.071	0.086	0.083	0.084	0.104	0.103	0.045	0.132	0.142	Equation (17)
Initial number of moles	1.897×10^{-4}	2.526×10^{-4}	2.02×10^{-4}	2.781×10^{-4}	2.147×10^{-4}	2.527×10^{-4}	1.604×10^{-4}	2.071×10^{-4}	2.408×10^{-4}	Equation (21)
Final number of moles	1.958×10^{-4}	2.511×10^{-4}	2.033×10^{-4}	2.877×10^{-4}	2.279×10^{-4}	5.878×10^{-4}	2.876×10^{-4}	2.684×10^{-4}	2.875×10^{-4}	Equation (22)
Δmoles	6.023×10^{-6}	-1.485×10^{-6}	1.283×10^{-6}	9.51×10^{-6}	1.322×10^{-5}	3.35×10^{-4}	1.272×10^{-4}	6.133×10^{-5}	4.671×10^{-5}	Equation (23)
ΔP_{moles} (mmHg)	0.66	−0.15	0.15	0.84	1.89	40.22	10.57	11.49	8.05	Equation (24)
ΔP_{Temp} (mmHg)	1.2	1.46	1.42	1.42	1.78	1.76	0.77	2.25	2.41	Equation (25)
Calculated ΔP (mmHg)	**1.86**	**1.31**	**1.57**	**2.26**	**3.66**	**41.98**	**11.35**	**13.74**	**10.45**	Equation (20)
Initial inflation gas within cuff	Nitrous Oxide/Oxygen	Nitrous Oxide/Oxygen	Nitrous Oxide/Oxygen	Nitrous Oxide/Oxygen	Nitrous Oxide/Oxygen	Room Air	Room Air	Room Air	Room Air	
Reference	*	*	*	*	*	**	***	***	***	

Other parameters, which have also been calculated, are referenced to their equation of origin. Adapted and compiled from: *Karasawa et al. (2001), **Revenäs et al. (1976), ***Raeder et al. (1985).

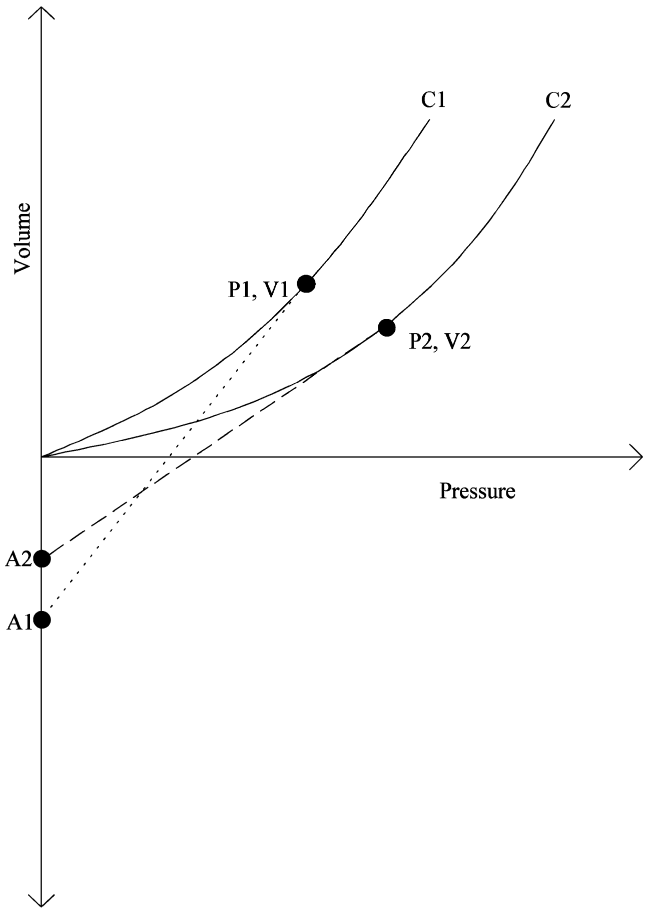

Fig. 2. C1 and C2 represent two different tracheal tube cuff compliance curves. Compliance is defined as the slope of these curves or dV/dP. Note that dV/dP, at point (P1, V1) is greater than dV/dP at point (P2, V2). Points A1 and A2 are the constants of integration associated with points (P1, V1) and (P2, V2) respectively. Furthermore, point (P1, V1) is associated with both an increased compliance and a constant of integration, point A1, which has a greater negative value than point A2.

Static model development

The modified ideal gas equation

The ideal gas equation has to be modified to accommodate changes in cuff volume as a function of pressure. Substituting the linear volume Equation (2) into the ideal gas equation yields:

$$P(CP + A) = nRT. \tag{5}$$

Expanding and rearranging Equation (5) generates the quadratic:

$$CP^2 + AP - nRT = 0. \tag{6}$$

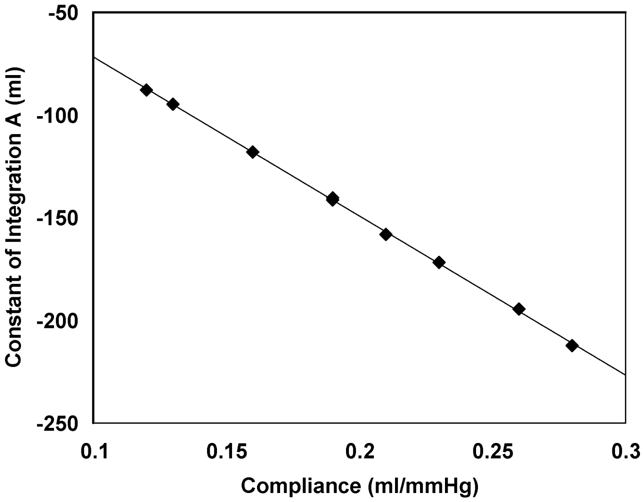

Fig. 3. The constant of integration, A, as a function of compliance. Data were calculated from each cuff in Table 1. Cuffs with greater compliance had associated constants of integration with greater negative values.

Therefore, pressure, P, can be expressed as:

$$P = \frac{-A + \sqrt{A^2 + 4nRTC}}{2C}. \tag{7}$$

Clearly, only the positive square root will yield a meaningful value for pressure. Using upper-limit clinical values for compliance (0.28 ml/mm Hg), number of moles (1.61×10^{-4}), and the constant of integration (−212.3 ml), this function is shown graphically in Figure 4(a). Note its "linear appearance" over the clinically relevant range of cuff gas temperature (18 through 37 °C). In addition, these values correspond to an initial cuff pressure of 11.7 mm Hg and a volume of 3.8 ml.

Using these same values as above, but keeping temperature fixed and increasing only the net number of moles diffusing into the cuff, a much greater increase in cuff pressure is noted. This is shown graphically in Figure 4(b). As in Figure 4(a), Figure 4(b) also has a "linear appearance." Both Figures 4(a) and (b) were derived using Equation (7). This "linear appearance" allows dP/dT and dP/dn to be approximated as constants.

Furthermore, this example is numerically based on relatively large values for compliance, the constant of integration, and net number of moles. Owing to the square root function in Equation (7), these larger values would graphically demonstrate nonlinearity better than smaller values.

This static model does not take into account simultaneous time-varying effects. These would include changes in the net molar quantity of cuff gas occurring *with* changes in cuff temperature. A dynamic model will allow for this.

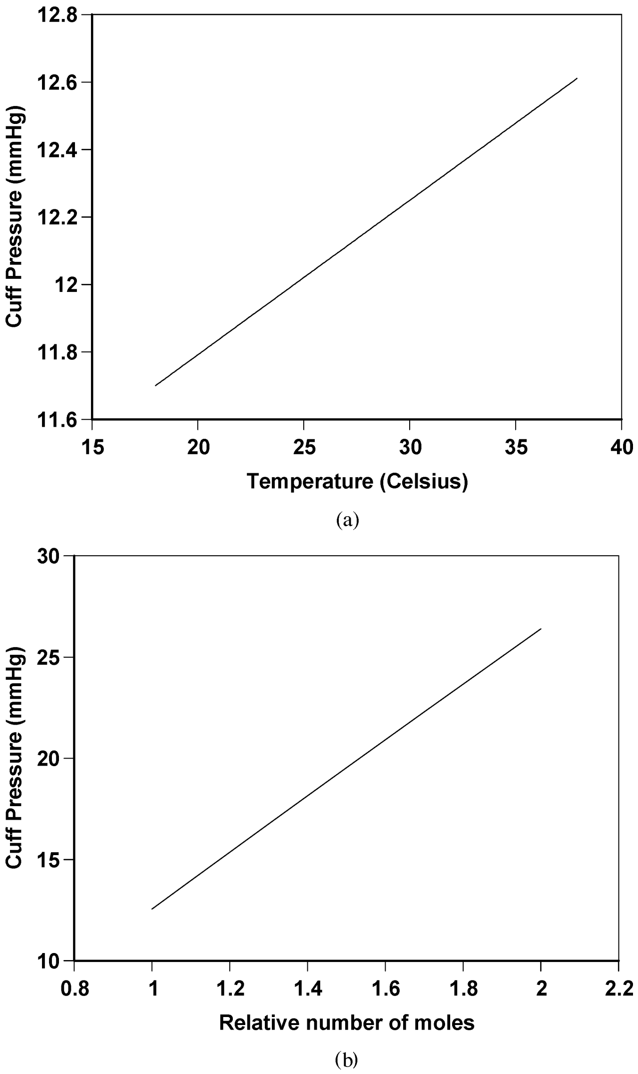

Fig. 4. (a, b) A small but significant increase in cuff pressure occurs as a function of increasing temperature. A much greater increase in this pressure occurs with increases in the molar quantity of gas within the cuff. Both of these effects have a "linear appearance" when examined graphically. Therefore, their respective slopes, dP/dT and dP/dn, can be approximated as constants.

Determining dP/dT and dP/dn

The change in cuff pressure with respect to temperature, dP/dT, can be determined by differentiating Equation (7):

$$\frac{dP}{dT} = \frac{d}{dT}\left[\frac{-A+\sqrt{A^2+4nRTC}}{2C}\right] = \frac{nR}{\sqrt{A^2+4nRTC}}. \tag{8}$$

Substituting Equation (7) back into (8) yields a simplified form of the derivative:

$$\frac{dP}{dT} = \frac{nR}{[2PC+A]}. \tag{9}$$

Equation (9) can also be obtained by implicitly differentiating Equation (6):

$$2CP\,dP + A\,dP - nR\,dT = 0 \tag{10}$$

$$\frac{dP}{dT}[2CP + A] = nR \tag{11}$$

$$\frac{dP}{dT} = \frac{nR}{[2CP + A]}. \tag{12}$$

Similarly, dP/dn can be determined as:

$$\frac{dP}{dn} = \frac{RT}{[2CP + A]}. \tag{13}$$

As shown in Appendix A, both dP/dT and dP/dn can also be derived based on a technique using approximations. Furthermore, it is instructive to examine how dP/dT and dP/dn are expressed with respect to elastance rather than compliance. This is shown in Appendix B.

For a given absolute pressure, the least amount of gas, or smallest number of moles of gas, with a cuff with the greatest possible compliance, will yield a lower dP/dT. However, since temperature is expressed in absolute terms, clinical changes in dP/dn tend to be almost strictly a function of compliance. Figures 5(a) and (b) show how dP/dT and dP/dn decrease with increasing compliance. These values were calculated from the clinical data shown in Table 1. Both dP/dT and dP/dn form the "cornerstones" of the physical behavior, of the tracheal cuff, to changes in temperature and total gas content.

Recall that together both compliance, C, and the constant of integration, A, describe the cuff's volume-pressure relationship within the "numerical proximity" of its initial inflated state. Not surprisingly, these factors, with the initial pressure, occur in the denominators of both dP/dn and dP/dT.

Comparing dP/dn to dP/dT

A comparison, of the effects of diffusion on cuff pressure, to those of temperature changes, can be made by examining the ratio:

$$\frac{\left(\frac{dP}{dn}\right)}{\left(\frac{dP}{dT}\right)} = \frac{\frac{RT}{[2CP+A]}}{\frac{nR}{[2CP+A]}} = \frac{T}{n}. \tag{14}$$

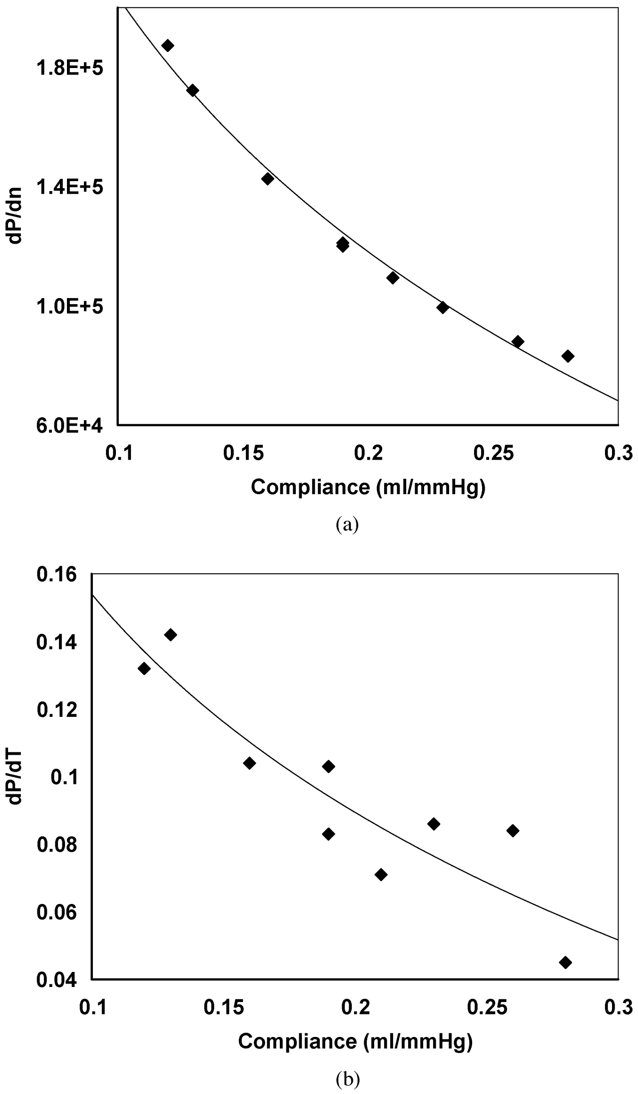

Fig. 5. (a, b) dP/dn and dP/dT, as a function of compliance, were calculated from each of the specific tracheal tube cuffs in Table 1.

With $T \approx 300\,^{\circ}$K and $n \approx 1.0 \times 10^{-4}$ moles. Therefore, dP/dn is much greater than dP/dT. Furthermore, this comparative ratio is independent of both cuff pressure, compliance, and the constant of integration. Consequently, under routine clinical conditions, cuff pressure is influenced far more by diffusion than by changes in temperature. Studies have confirmed this large increase in cuff pressure from the diffusion of nitrous oxide [13, 14].

Table 2 and Figure 6 show how tracheal tube cuffs, which had been inflated with room air, had an increase in pressure over 2 h. However, those cuffs, which had been inflated with combinations of nitrous oxide and oxygen, tended not to have had this magnitude of an increase in pressure. It should be noted that both groups of tracheal tube cuffs had been exposed to anesthetic gas mixtures which consisted of inhalational anesthetic, oxygen, *and nitrous oxide* [20].

Clearly, the nitrous oxide concentration gradient, occurring from the anesthetic gas mixture outside the cuff, to that within the cuff, is the primary factor in the change in cuff pressure over time. Therefore, the diffusion of nitrous oxide, into the cuff, is greater than the diffusion of the room air out of the cuff.

The effect of different mean airway pressures, on this diffusion process, has not yet been examined.

Furthermore, several studies have shown significant decreases in cuff pressure with cardiopulmonary bypass and hypothermia. With rewarming, cuff pressure tends to return to baseline levels. It should be noted that these particular studies were done following a median sternotomy incision. Thus, tracheal transmural pressure may have been reduced because of this. In addition, the mechanical properties of the trachea may change under these circumstances [15–17].

Development of a dynamic model

A time-varying model, which would represent simultaneous changes in tracheal tube cuff pressure, from both diffusion and temperature, could be based on the well-known chain rule:

$$\frac{dP}{dt} = \frac{dP}{dn}\frac{dn}{dt} + \frac{dP}{dT}\frac{dT}{dt}. \tag{15}$$

We are examining dP/dn and dP/dT occurring within small ranges of the initial pressure, P_i, initial temperature, T_i, and initial molar quantity of gas within the cuff, n_i. Therefore, numerical values, for dP/dn and dP/dT, could be based on these initial values. This would simplify the dynamic model and allow both dP/dn and dP/dT to be expressed as constants. dP/dn and dP/dT are then approximated as:

$$\frac{dP}{dn} \approx \frac{RT_i}{[2CP_i + A]} \tag{16}$$

$$\frac{dP}{dT} \approx \frac{n_i R}{[2CP_i + A]}. \tag{17}$$

Moreover, the "linear appearance" of *cuff pressure vs. temperature* as well as *cuff pressure vs. number of moles*, in Figure 4(a) and (b), substantiates that dP/dT and dP/dn can be accurately expressed as constants.

In addition, graphical analysis, of the clinical data, substantiate that dP/dT and dP/dn vary inversely with compliance. This is shown in Figure 5(a) and (b).

Table 2. *Pressure values inside the cuffs of tracheal tubes during general anesthesia. (Mean values in each group in mmHg). Abbreviations: National Catheter Company (NCC) and reverse angle endotracheal (rae). Each tracheal tube cuff was inflated with either air or nitrous oxide/oxygen*

		Time (min.)											
Groups	Start	10	20	30	40	50	60	70	80	90	100	110	120
NCC rae, air	11.7	13.4	15.5	18.0	20.0	21.2	22.3	23.4	24.5	25.2	25.8	25.3	23.4
NCC plain, air	12.5	14.3	15.6	17.1	19.0	19.5	20.4	21.5	22.3	23.8	24.5	26.3	26.5
Portex Blue Line, air	12.3	14.9	16.6	17.4	19.2	19.1	19.9	19.7	20.1	22.7	23.1	22.4	23.2
NCC, nitrous oxide/oxygen	12.8	12.1	12.5	13.0	12.9	12.9	12.6	12.9	13.0	13.5	14.0	13.8	13.7
NCC plain, nitrous oxide/oxygen	13.1	11.9	12.5	13.1	12.8	12.7	12.7	13.4	15.4	15.1	14.6	15.7	15.1
Portex Blue Line, nitrous oxide/oxygen	13.2	12.9	13.1	13.1	13.5	13.9	13.8	13.8	13.9	14.5	14.1	13.7	13.5

Note that nitrous oxide/oxygen filled cuffs exhibited a small increase in pressure which was probably attributed from increases in temperature. However, in most cases, small amounts of nitrous oxide apparently had also diffused into the cuffs from the anesthetic gas mixture. These parameters are calculated in Table 1. Air-filled tracheal cuffs exhibited a significantly greater increase in pressure from a greater amount of nitrous oxide which had diffused into the cuff. These data are shown graphically in Figure 6. Reproduced from Raeder et al. (1985).

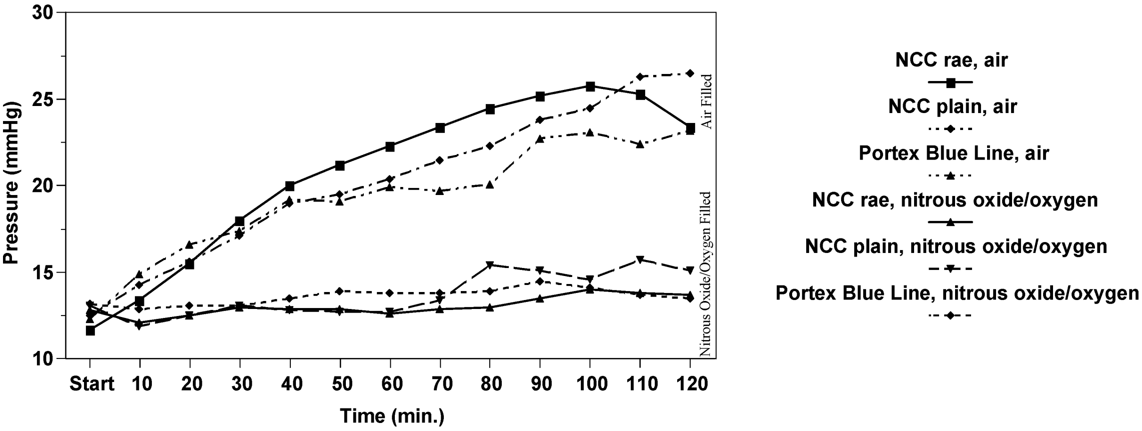

Fig. 6. *Data from Table 2 shown graphically. Note how air-filled tracheal cuffs greatly increased in pressure whereas nitrous oxide/oxygen filled cuffs exhibited a minimal increase. This pressure increase appears to have a temperature-related component. Adapted from Raeder et al. (1985).*

Determining the net molar flow rate

Pressure, within the tracheal tube cuff, can be easily measured. Likewise, cuff temperature can be estimated with a readily-available esophageal temperature gauge. It should be noted that the trachea and the esophagus are anatomically adjacent.

The combined diffusion of nitrous oxide into the cuff and air out of the cuff or "net molar flow rate" can then be determined:

$$\frac{\frac{dP}{dt} - \left(\frac{dP}{dT}\frac{dT}{dt}\right)}{\frac{dP}{dn}} = \frac{dn}{dt}. \tag{18}$$

Therefore, the net number of moles, from diffusion into the cuff, could also be calculated in real-time with clinically-obtained data:

$$\frac{\Delta P - \left(\frac{dP}{dT}\right)\Delta T}{\left(\frac{dP}{dn}\right)} = n. \tag{19}$$

Methods

It should be noted that, in analyzing these compiled studies, it was assumed that room temperature gases (20 °C) were used to inflate the cuffs and that the temperature of these gases increased to body temperature (37 °C).

In several studies, when nitrous oxide/oxgen had been used to inflate the cuffs, volume changes were not always detectable. However, associated pressure changes were. This was most likely do to errors in experimental measurement. Only those cuffs, which had both measurable pressure and volume changes, were included in this analysis [19–21].

Calculations for ΔP were made by modifying Equation (15):

$$\Delta P = \frac{dP}{dn}\Delta n + \frac{dP}{dT}\Delta T. \tag{20}$$

Where ΔT was estimated to be body temperature minus room air temperature: $T_f - T_i = 37 - 20 = 17\,^\circ\mathrm{C}$. This assumed that room temperature gases were injected into the cuff which then heated to body temperature. Note that temperature *differences* are numerically equivalent in both the Celsius and Kelvin scales. In addition, Δn was calculated based on:

$$n_i = \frac{P_i V_i}{RT_i} \tag{21}$$

$$n_f = \frac{P_f V_f}{RT_f} \tag{22}$$

$$\Delta n = n_f - n_i = \left[\frac{P_f V_f}{RT_f} - \frac{P_i V_i}{RT_i}\right] = \frac{1}{R}\left[\frac{P_f V_f}{T_f} - \frac{P_i V_i}{T_i}\right]. \tag{23}$$

ΔP was then determined based upon dP/dn and dP/dT values from Equations (16) and (17). These results, based on clinical data, are shown in Table 1.

Equation (20) was also used to assess the differences between molar and thermal contributions to changes in cuff pressure:

$$\Delta P_{\text{moles}} = \frac{dP}{dn}\Delta n \tag{24}$$

$$\Delta P_{\text{Temp}} = \frac{dP}{dT}\Delta T. \tag{25}$$

Therefore, Equation (20) can also be expressed as:

$$\Delta P = \Delta P_{\text{moles}} + \Delta P_{\text{Temp}}. \tag{26}$$

Analytical results: Comparing calculated ΔP to measured ΔP

The results of these calculations, based on the clinical data in Table 1, support that overall cuff pressure changes can be modeled as a combination of the individual effects of diffusion and temperature.

Specifically, an increase in the net molar contents, of the tracheal tube cuffs, was noted in all but the *Hi-Contour* cuff. In this particular case, a small leak may have occurred. Specifically, only a leak, or diffusion of gases out of the cuff, would explain the negative change in the number of moles of gas within the cuff.

Nonetheless, the sum of the calculated values, for ΔP_{moles} and ΔP_{Temp}, produced a realistic and reasonable model of the overall pressure change that occurred within each tracheal tube cuff.

ΔP_{moles} was greatest for those cuffs filled with air as opposed to those filled with nitrous oxide. Furthermore, in all cases, ΔP_{Temp} was a small but noticeable component to the total change in cuff pressure.

DISCUSSION

Clinicians and engineers under appreciate the dynamics of the tracheal tube cuff. Not only does this device assist in providing the basis for achieving positive pressure ventilatory support, but it is an important barrier to infection during general anesthesia and critical care. This is achieved from the seal, created by the cuff against the tracheal wall, which helps to prevent aspiration.

However, significant morbidity and mortality exist, with excessive cuff pressure, and with long-term intubation without excessive cuff pressure [1–12].

A model of differential tracheal tube cuff pressure has been presented. Using this model, the effects of differences in cuff permeability, surface area, and thickness would be manifested as different changes in measured Δn. Whereas the cuff's initial conditions and volume-pressure relationship form both $\mathrm{d}P/\mathrm{d}n$ and $\mathrm{d}P/\mathrm{d}T$. Clearly, when compared with $\mathrm{d}P/\mathrm{d}T$, $\mathrm{d}P/\mathrm{d}n$ is the leading component in modeling changes in cuff pressure.

Nonetheless, a small but consistent increase in cuff pressure apparently occurs with the warming of room temperature gases injected into the cuff. This apparently happens regardless of the nature of the cuff inflation gases. Therefore, temperature-related pressure increases appear to be independent of any diffusion process which might be taking place simultaneously.

Furthermore, using this model, these simultaneous changes, in cuff molar contents and temperature, can be independently assessed as they effect cuff pressure.

Additional parameters could also be examined in more detail with further model development. These would include: mean airway pressure within the trachea, cuff permeability with respect to individual gases, as well as cuff thickness and exposed surface area. Collectively, these parameters could be assessed [24]:

$$\Delta n = J_o\left[\frac{A}{h}\right][P_{\bar{a}} - P_i]\Delta t. \tag{27}$$

Where J_o is the "overall" cuff permeability coefficient whereas A and h represent exposed cuff surface area and thickness respectively. $P_{\bar{a}}$ represents mean airway pressure outside the cuff.

The specific effects of each gas could also be examined:

$$\Delta n = \sum_k J_k \left[\frac{A}{h}\right]\left[P_{\bar{a}_k} - P_{i_k}\right]\Delta t \tag{28}$$

In the above equation, J_k represents the cuff permeability coefficient for each individual gas, k. In addition, $P_{\bar{a}_k}$ is the average partial pressure of each gas within the trachea and P_{i_k} is the initial partial pressure of each gas within the cuff.

CONCLUSION

The effect of nitrous oxide, diffusing into an air-filled cuff, remains a significant issue. Too often, clinicians will fill the cuff with air and administer nitrous oxide, with other inhalational anesthetics, without further assessing increases in cuff pressure. Similarly, no "warning" is given by manufactures that this condition can occur. Yet, patient safety is the goal of both the clinical and engineering communities.

In addition, clinicians will sometimes forego the use of cuffed tracheal tubes for fear that they may damage the trachea. This is particularly common in pediatric anesthesia. Yet, it should be realized that the pressure within the cuff, and not the cuff itself, is the causative factor in cuff-related tracheal pathology. Consequently, the potential clinical benefits of the cuff are then lost.

Based on this analytical assessment of existing data, a small increase in cuff pressure apparently occurs with the warming of room air gases, injected into the cuff, regardless of the nature of these gases. Depending on clinical circumstances, this temperature-related increase in cuff pressure may be significant. Examples would include hypothermic patients, who are intubated in the field and then require rewarming, or febrile patients.

This model enables the effects of temperature and diffusion to be rapidly and easily modeled and assessed. The significant pathological effects of excessive cuff pressure should therefore be better recognized and appreciated in future tracheal tube cuff design, development, and clinical use.

APPENDIX A

Determining dP/dT and dP/dn using approximations

Equations (16) and (17) can also be derived using an approximate technique. These approximations may be useful in clinical assessments of tracheal tube cuff pressure.

Initially, n_1 and n_2 are defined in order to ultimately determine Δn. These are associated with P_1 and P_2 and ΔP.

$$\frac{\left[CP_1^2 + AP_1\right]}{RT} = n_1 \tag{A.1}$$

$$\frac{\left[CP_2^2 + AP_2\right]}{RT} = n_2 \tag{A.2}$$

Therefore Δn is:

$$\begin{aligned}\Delta n &= n_2 - n_1\\ &= \frac{1}{RT}\left[\left(CP_2^2 + AP_2\right) - \left(CP_1^2 + AP_1\right)\right].\end{aligned} \tag{A.3}$$

Rearranging Equation (A.3) yields:

$$\Delta n = \frac{1}{RT}\left[\left(CP_2^2 - CP_1^2\right) + \left(AP_2 - AP_1\right)\right]. \tag{A.4}$$

Realizing that $CP_2^2 - CP_1^2$ can be expressed as *a difference of two squares*:

$$\Delta n = \frac{(P_2 - P_1)}{RT}[C(P_2 + P_1) + A]. \tag{A.5}$$

Defining $\Delta P = P_2 - P_1$

$$\Delta n = \frac{\Delta P}{RT}[C(P_2 + P_1) + A]. \tag{A.6}$$

dP/dn is then approximated as $\Delta P/\Delta n$:

$$\frac{dP}{dn} \approx \frac{\Delta P}{\Delta n} \approx \frac{RT}{[C(P_2 + P_1) + A]} \approx \frac{RT_i}{[2CP_i + A]} \tag{A.7}$$

Equation (A.7) is equivalent to Equation (16) as $(P_2 + P_1) \approx 2P_i$.

Similarly, dP/dT can be found as:

$$\frac{dP}{dT} \approx \frac{\Delta P}{\Delta T} \approx \frac{nR}{[C(P_2 + P_1) + A]} \approx \frac{n_i R}{[2CP_i + A]}. \tag{A.8}$$

APPENDIX B

Expressing dP/dT and dP/dn as a function of elastance

Realizing that elastance, E, is the multiplicative inverse of compliance, C:

$$C = \frac{dV}{dP} \quad \text{and} \quad E = \frac{dP}{dV} = \frac{1}{C}. \tag{B.1}$$

Therefore:

$$dP = E\,dV. \tag{B.2}$$

Integrating yields the constant of integration B:

$$P = \int dP = E \int dV = EV + B \tag{B.3}$$

$$P = EV + B. \tag{B.4}$$

Solving for V:

$$\frac{P - B}{E} = V. \tag{B.5}$$

The ideal gas equation:

$$PV = nRT. \tag{B.6}$$

Substituting (B.5) into (B.6):

$$P\left[\frac{P - B}{E}\right] = nRT. \tag{B.7}$$

Expanding (B.7):

$$\frac{P^2}{E} - \frac{B}{E}P - nRT = 0. \tag{B.8}$$

Solving the above quadradric yields:

$$P = \frac{B + \sqrt{B^2 + nRTE}}{2}. \tag{B.9}$$

Implicitly differentiating (B.8) yields:

$$\frac{2P}{E}dP - \frac{B}{E}dP - RTdn = 0. \tag{B.10}$$

Collecting terms:

$$\frac{dP}{dn}\left[\frac{2P}{E} - \frac{B}{E}\right] - RT = 0. \tag{B.11}$$

Rearranging:

$$\frac{dP}{dn} = \frac{RT}{\left(\frac{2P}{E} - \frac{B}{E}\right)}. \tag{B.12}$$

Collecting terms and simplifying:

$$\frac{dP}{dn} = E \cdot \frac{RT}{(2P - B)}. \tag{B.13}$$

Similarly:

$$\frac{dP}{dT} = E \cdot \frac{nR}{(2P - B)}. \tag{B.14}$$

Therefore, dP/dn and dP/dT are both directly proportional to elastance, E.

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